

Robust Data-driven Control for Cooperative Slung Non-Rigid Payload Drone System

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Introduction

Background: Global warming has potentially resulted in an increase in the frequency and severity of fire risks across Canada. The main focus of fire-related research is expected to target the capability of preparation for fire hazards and the reduction of both the risk and the severity of the consequences of fire hazards.

Challenge: Traditional control techniques provide either non-functional or conservative commands for non-rigid payload situations, resulting in instability of the system or trajectory tracking errors with degraded performance.

Contribution: This work proposed a sloshing modeling method, integrating with the UAV dynamic models derived using Kane's method. A robust MPC is introduced to guarantee system stability and trajectory tracking accuracy of the payload.

System Illustration

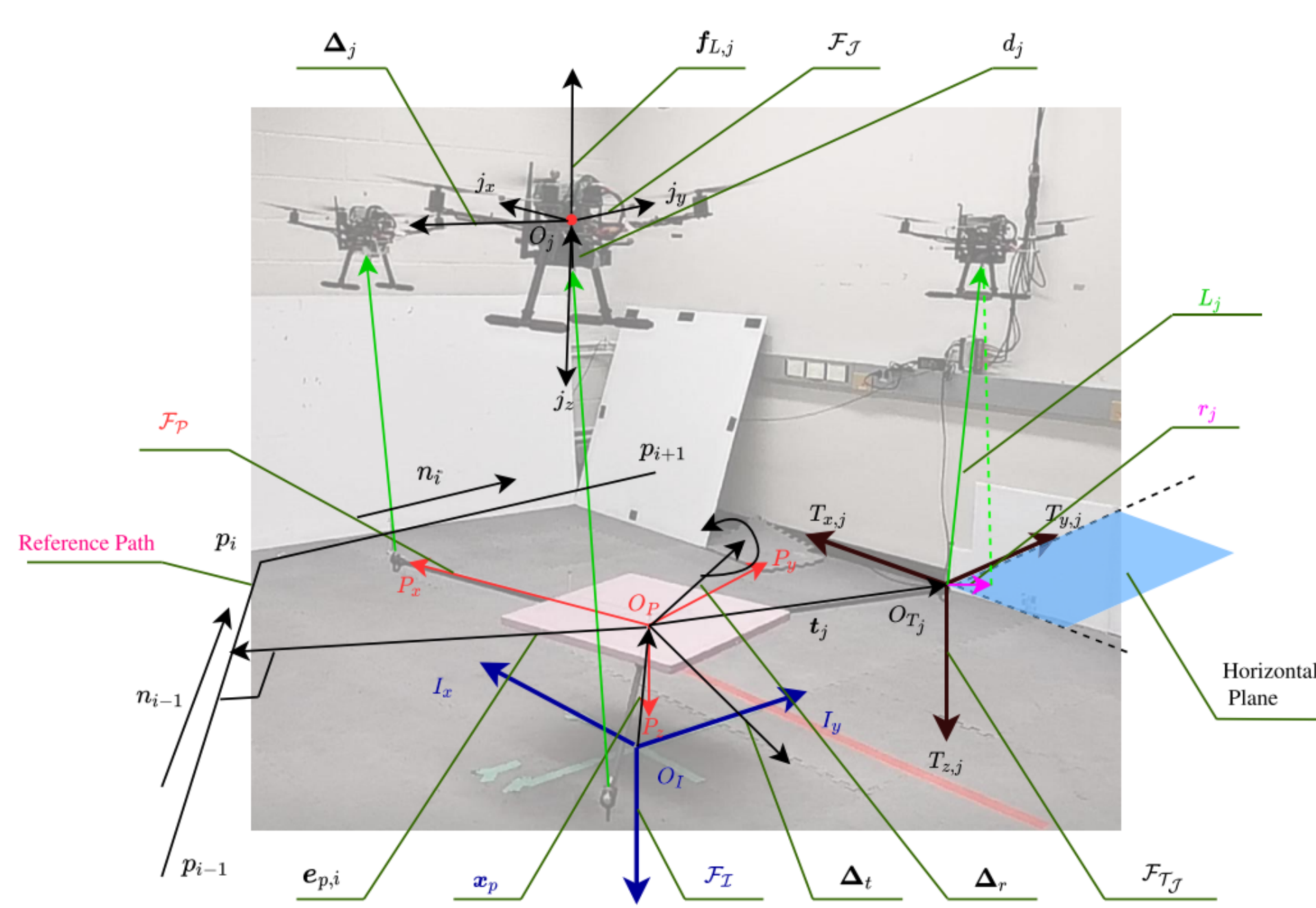


Figure 1: Slung Payload Model [1]

[1] L. Qian, H. Liu. "Robust Control Study for Tethered Payload Transportation Using Multiple Quadrotors," in Journal of Guidance, Control, and Dynamics, vol. 45, no. 3, pp. 434-452, 2022.

Sloshing Model

Liquid Container Illustration

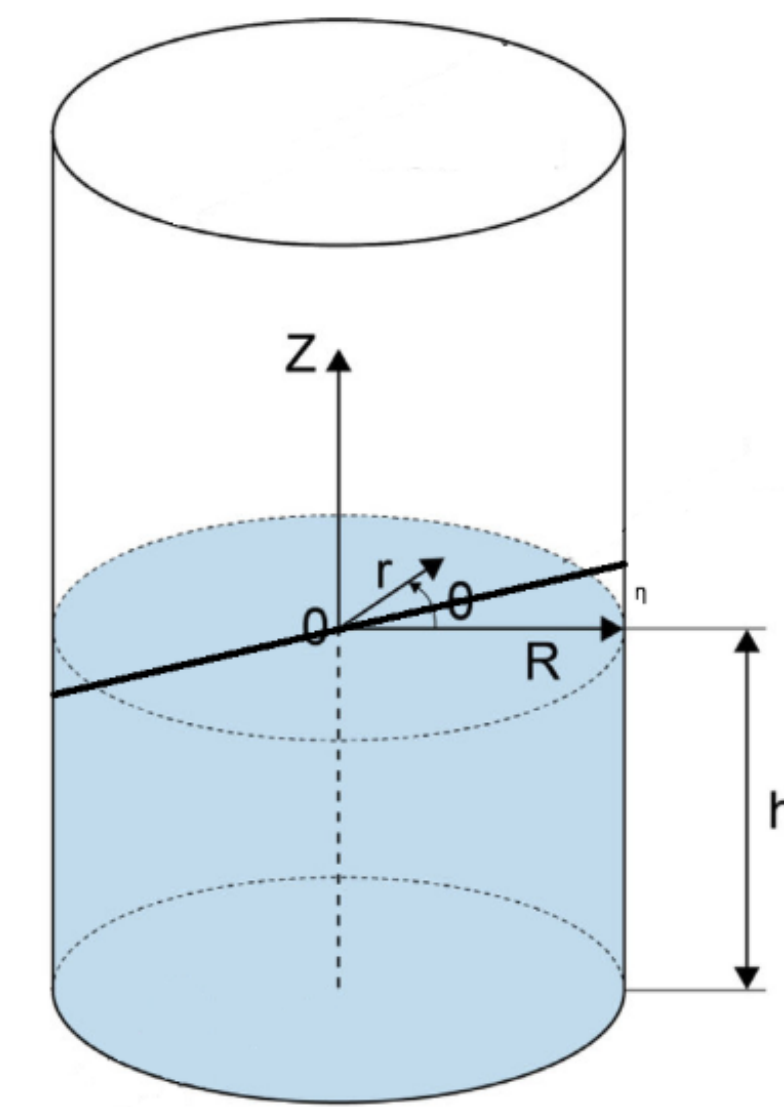


Figure 2: Liquid Container

Assumptions: Single-dimension activations; irrotational liquid; dominant mode only

Governing equation:

$$\nabla^2 \tilde{\Phi} = 0 \quad (1)$$

Boundary conditions:

$$\left. \frac{\partial \tilde{\Phi}}{\partial r} \right|_{r=R} = 0, \quad \left. \frac{\partial \tilde{\Phi}}{\partial z} \right|_{z=-h_f} = 0, \quad \frac{\partial \eta}{\partial t} = \left. \frac{\partial \tilde{\Phi}}{\partial z} \right|_{z=0} \quad (2)$$

$$\left. \frac{\partial \tilde{\Phi}}{\partial t} \right|_{z=0} + (a_{Fx} + g\gamma)r \cos \theta + g_{\text{eff}}\eta = 0 \quad (3)$$

Velocity potential:

$$\tilde{\Phi} = \sum_{n=1}^{\infty} \phi_n(r, \theta, z) \dot{q}_n(t) \quad (4)$$

with mode shape

$$\phi_n = J_1(k_{1n}r) \cosh \left[\frac{\lambda_{1n}}{r} (z + h_f) \right] \cos(\theta) \quad (5)$$

Free surface elevation: (Dominant mode)

$$\eta(r, \theta, t) = J_1 \left(\frac{\lambda_{11}r}{R} \right) \frac{\lambda_{11}}{R} \sinh \left(\frac{\lambda_{11}h_f}{R} \right) \cos(\theta) q(t) \quad (6)$$

Galerkin Projection:

The **residual** is defined as follows, which is expected to vanish:

$$R(r, \theta) = \sum_{n=1}^{\infty} J_1(k_{1n}r) \cos(\theta) \ddot{q}_n + g_{\text{eff}} \sum_{n=1}^{\infty} k_{1n} \tanh(k_{1n}h) J_1(k_{1n}r) \cos(\theta) q_n + (a_{Fx} + g\gamma)r \cos(\theta) \quad (7)$$

Sloshing Model

The **weight** assigned to the residual is given as:

$$W_p(r, \theta) = J_1(k_{1p}r) \cos(\theta) \quad (8)$$

Integrate over the circular free surface,

$$\int_0^{2\pi} \int_0^R R(r, \theta) W_p(r, \theta) r dr d\theta = 0 \quad (9)$$

Thus:

$$N_p \ddot{q}_p + N_p g_{\text{eff}} k_{1p} \tanh(k_{1p}h) q_p = -F_p(a_{Fx} + g\gamma) \quad (10)$$

Comparing Eq. (10) with the standard second-order oscillator,

$$\ddot{q} + \omega^2 q = f \quad (11)$$

Gives the natural frequency of the p -th sloshing mode,

$$\omega_{1p}^2 = g_{\text{eff}} k_{1p} \tanh(k_{1p}h) \quad (12)$$

Control Design

Prediction Model:

$$X = \Phi x_k + \Gamma U + \Gamma_d D \quad (13)$$

Optimization Problem:

$$\min J$$

$$\begin{aligned} \text{subject to } & z_{i+1} = A_d z_i + B_d u_i, \\ & z_i \in \mathcal{X} \ominus \mathcal{E}, \\ & u_i \in \mathcal{U} \ominus K\mathcal{E}, \\ & z_N \in \mathcal{X}_f, \end{aligned} \quad (14)$$

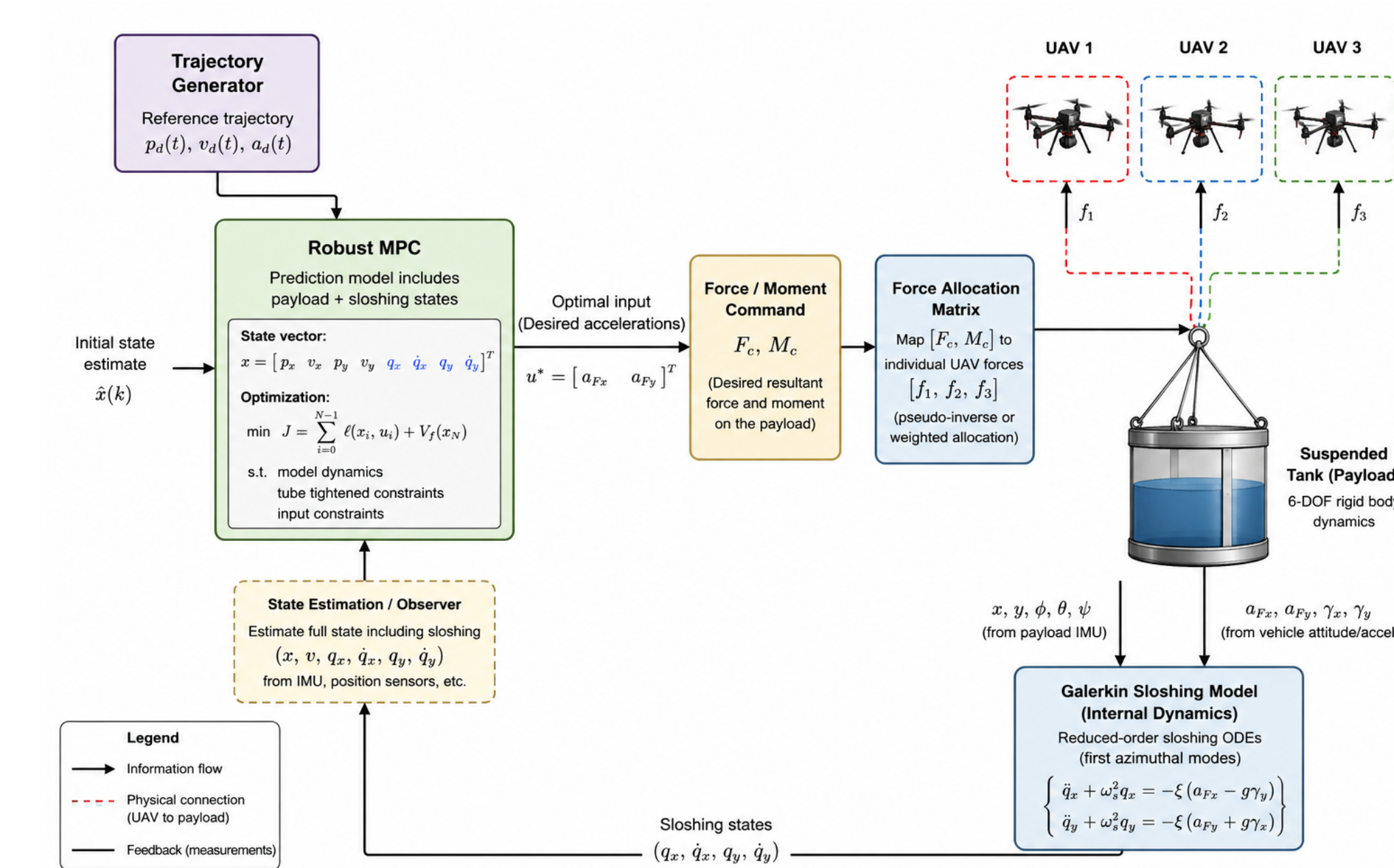


Figure 3: Control Diagram

Results & Conclusion

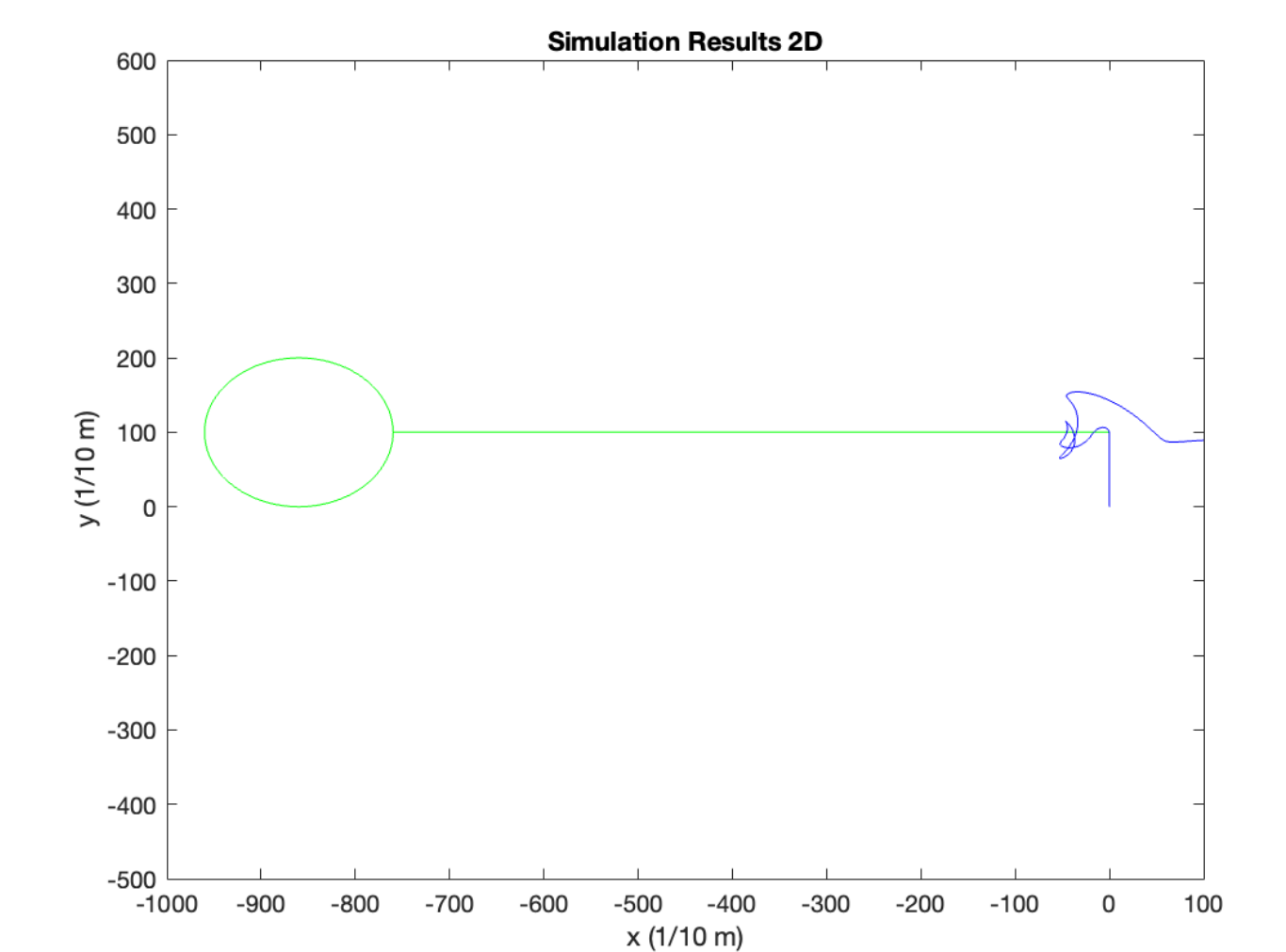


Figure 4: PID Control without Sloshing Consideration

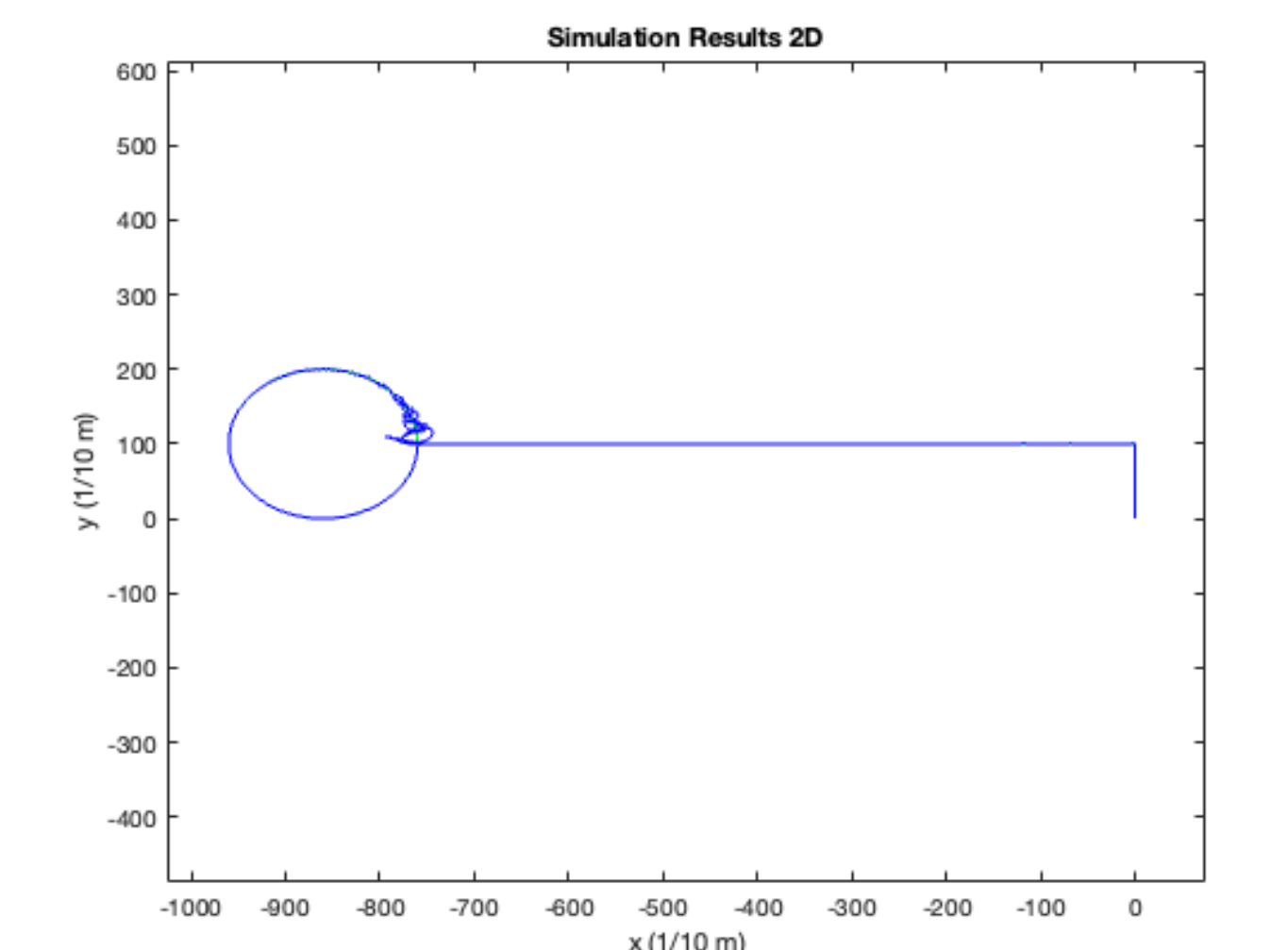


Figure 5: MPC Control with Sloshing Model

The proposed method is being evaluated with an adaptive PID control system applied on the dynamic model of the slung payload system without considering the sloshing effects. The results demonstrate the feasibility of the current design through an average trajectory tracking error within 0.1 m.

Future Works

The main focus of future works will be a generalized model and control for all non-rigid payload cases. The following factors are to be considered in the next steps of this research:

- 3-dimensional activations of sloshing
- Rotational liquid
- Adjustable cable length and elastic cable